UNIVERSIDAD NACIONAL DE INGENIERÍA

FACULTAD DE INGENIERÍA ECONÓMICA Y CIENCIAS SOCIALES

4º CALIFICADA DE ÁLGEBRA LINEAL

1.- Determine el valor de verdad de las siguientes proposiciones, **justificando con argumentos teóricos su respuesta.**

Sabiendo que los vectores **a, b y c** están en R^3

$a)$ $(a+b+c)\bullet(a-2b+2c)\times(4a+b+5c)\neq 0$

b) $(a+b)\bullet b\times(a+c)=-[abc]$

c) $(a-b)\bullet(a-b-c)\times(a+2b-c)=3[abc]$

d) $a\bullet b\times(c+ra+sb)\neq[abc]$ $,\forall\ r,s\in\mathbb{R}$

$2.-$ *Sean las rectas* L_1 *y* L_2 *que* se c*ruzan ortogonalmente, donde*

$L_1=\{B+t(1,1,0),\mathrm{t}\in\mathbb{R}\}$ y $L_2=\{C+r(-3,3,1),\mathrm{r}\in\mathbb{R}\},B=(-4,7,1)C=(3,1,2)$

La distancia mínima entre L_1 *y* L_2 *es el segmento* $\overline{AD}$ $(\mathrm{A}\in L_1,\mathrm{D}\in L_2)$

P es un plano paralelo a las rectas $\mathrm{L}_1 y \mathrm{L}_2$ que pasa por el punto medio de $\overline{AD}$,

$\mathrm{M} = \mathrm{P}\cap\mathrm{L}_{AC}$ y $\mathrm{N} = \mathrm{P}\cap\mathrm{L}_{BD}$.Calcule $\left|\overline{MN}\right|$.

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3.– a) L es una recta ortogonal a las rectas:

$$L_1 : \frac{x+6}{2} = y-1 = \frac{z+1}{-1}$$

$$L_2 : x-3 = \frac{y}{2},\ z = 2$$

Determine la ecuación del plano P que pasa por el punto $(0,0,-2)$ y que contiene a la recta L.

b) Analice las siguientes proposiciones y justifique su respuesta con haciendo uso de argumentos teóricos.

i) $(a\times b)\times(b\times c)\bullet(c\times a) = \left[abc\right]^2$

ii) Si $\operatorname{Proy}_b a = (4,4,-8)$ y $\operatorname{Proy}_a b = (8,-4,-4)$ entonces $a\bullet b = 64$.

4.– Dado el triángulo ABC, donde $C = (-5,14,-3)$, la bisectriz interior del $\measuredangle ABC$ es $L_1 : x-1 = \frac{y-2}{-3} = \frac{z+7}{-8}$ y la mediana trazada desde A al lado $\overline{BC}$ es $L_2 = \left\{(-7,17,-9)+t(-5,9,-4),\ t\in\mathbb{R}\right\}$.

Determine los vértices del triángulo ABC.

EL PROFESOR

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4 PC ALGEBRA LINEAL

1 - 4
2 - 3,5
3 - 5
4 - 2,5
TP - 3

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FIRMA :

1.- Sabiendo que los vectores a, b y $c \in \mathbb{R}^3$

a) $(a+b+c) \cdot [(a-2b+2c) \times (4a+b+5c)] \neq 0$ (F)

Sean a, b y $c \in \mathbb{R}^3$

Entonces

$$\left.\begin{array}{l} m = a+b+c \\ n = a-2b+2c \\ p = 4a+b+5c \end{array}\right\} \begin{array}{l} p-3m = 4a+b+5c-3a-3b-3c \\ \qquad = a-2b+2c \end{array}$$

$$p-3m = n$$

$\rightarrow \{m, n, p\}$ son linealmente dependientes

$\therefore [m \; n \; p] = 0$

La proposición es falsa.

b) $(a+b) \cdot b \times (a+c) = -[abc]$ (F)

$$\left.\begin{array}{l} m = a+b \\ n = b \\ p = a+c \end{array}\right\} [m \; n \; p] = \begin{vmatrix} a_1+b_1 & a_2+b_2 & a_3+b_3 \\ b_1 & b_2 & b_3 \\ a_1+c_1 & a_2+c_2 & a_3+c_3 \end{vmatrix}$$

$$[m \; n \; p] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ a_1+c_1 & a_2+c_2 & a_3+c_3 \end{vmatrix}$$

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ a_1+c_1 & a_2+c_2 & a_3+c_3 \end{vmatrix} \sim \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = [a\ b\ c]$$

$$\longrightarrow [m\ n\ p] = [a\ b\ c]$$

$$\therefore [(a+b)\ b\ (a+c)] = [a\ b\ c]$$

La proposición es falsa

c) $(a-b) \cdot (a-b-c) \times (a+2b-c) = 3[a\,b\,c]$ (V)

$$\left.\begin{array}{l} m = a-b \\ n = a-b-c \\ p = a+2b-c \end{array}\right\} [m\ n\ p] = \begin{vmatrix} a_1-b_1 & a_2-b_2 & a_3-b_3 \\ a_1-b_1-c_1 & a_2-b_2-c_2 & a_3-b_3-c_3 \\ a_1+2b_1-c_1 & a_2+2b_2-c_2 & a_3+2b_3-c_3 \end{vmatrix}$$

$$[m\ n\ p] = \begin{vmatrix} a_1-b_1 & a_2-b_2 & a_3-b_3 \\ -c_1 & -c_2 & -c_3 \\ 3b_1 & 3b_2 & 3b_3 \end{vmatrix}$$

$$[m\ n\ p] = -3\begin{vmatrix} a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$[m\ n\ p] = 3\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 3[a\ b\ c]$$

$$\longrightarrow [(a-b)\ (a-b-c)\ (a+2b-c)] = 3[a\ b\ c]$$

$\therefore$ La proposición es verdadera

d) $a \cdot b \times (c + ra + sb) \neq [a\ b\ c]$, $\forall\, r, s \in \mathbb{R}$ (F)

$$\left.\begin{array}{l} m = a \\ n = b \\ p = c + ra + sb \end{array}\right\} \quad p = c + mr + ns$$

$$\longrightarrow [m\ n\ p] = [a\ b\ c]$$

$$[a\ b\ (c + ra + sb)] = [a\ b\ c]$$

$\therefore$ La proposición es falsa

2.-

$L_1 = \{ (-4,7,1) + t(1,1,0),\ t \in \mathbb{R} \}$

$L_2 = \{ (3,1,2) + r(-3,3,1),\ r \in \mathbb{R} \}$

$L_1 \begin{cases} P_0 = (-4,7,1) \\ V_1 = (1,1,0) \end{cases}$ $\quad L_2 \begin{cases} Q_0 = (3,1,2) \\ V_2 = (-3,3,1) \end{cases}$

$A = (t-4;\ t+7,\ 1)$

$D = (3-3r,\ 3r+1,\ r+2)$

$\vec{AD} = D - A = (7-3r-t;\ 3r-t-6;\ r+1)$

PUNTO MEDIO DE AD: E

$$E = \left(\frac{t-3r-1}{2};\ \frac{t+3r+8}{2};\ \frac{r+3}{2} \right) \in \mathcal{P}$$

Vector normal de $\mathcal{P}$:

$N = V_1 \times V_2 = (1,1,0) \times (-3,3,1) = (1,-1,6)$

Como es $\vec{AD}$ es la distancia mínima de L_1 y L_2

$\to \vec{AD} \cdot V_1 = 0 \to 7-3r-t = 6+t-3r \ \ldots (i)$

$\to \vec{AD} \cdot V_2 = 0 \to -3(7-3r-t) + 3(3r-t-6) + r+1 = 0 \ldots (ii)$

De (i): $7 - t = 6 + t$

$$\boxed{\frac{1}{2} = t}$$

De (ii):

$$-21 + 9r + \cancel{3t} + 9r - \cancel{3t} - 18 + r + 1 = 0$$

$$19r - 38 = 0$$

$$\boxed{r = 2}$$

$$A = \left(\frac{1}{2} - 4 ; \frac{1}{2} + 7 ; 1\right) = \left(-\frac{7}{2} ; \frac{15}{2} ; 1\right)$$

$$D = \left(3 - 3(2) ; 3(2) + 1 ; 2 + 2\right) = \left(-3 ; 7 ; 4\right)$$

$$E = \left(-\frac{13}{4} ; \frac{29}{4} ; \frac{5}{2}\right)$$

Ecuación del Plano:

$$1\left(x + \frac{13}{4}\right) - 1\left(y - \frac{29}{4}\right) + 6\left(z - \frac{5}{2}\right) = 0$$

$$\frac{4x + 13}{4} - \frac{(4y - 29)}{4} + \frac{6(2z - 5)}{2} = 0$$

$$4x + 13 - 4y + 29 + 24z - 60 = 0$$

$$4x - 4y + 24z = 18$$

$$\mathcal{P} : 2x - 2y + 12z = 9$$

Hallando las rectas:

$$L_{AC} = \{ (3, 1, 2) + t(13, -13, 2) ; t \in \mathbb{R} \}$$

$$L_{BD} = \{ (-4, 7, 1) + r(1, 0, 3) , r \in \mathbb{R} \}$$

Hallando M:

$$2(13t+3)-2(-13t+1)+12(2t+2)=9$$

$$26t+6+26t-2+24t+24=0$$

$$52t+24t+28=0$$

$$76t=-28$$

$$t=\frac{-7}{19}$$

$$M=\left[13\left(\frac{-7}{19}\right)+3\ ;\ -13\left(\frac{-7}{19}\right)+1\ ;\ 2\left(\frac{-7}{19}\right)+2\right]$$

$$M=\left(\frac{-34}{19}\ ;\ \frac{110}{19}\ ;\ \frac{24}{19}\right)$$

Hallando N:

$$2(r-4)-2(7)+12(3r+1)=0$$

$$2r-8-14+36r+12=0$$

$$38r=10$$

$$r=\frac{5}{19}$$

$$N=\left[\frac{5}{19}-4\ ;\ 7\ ;\ 3\left(\frac{5}{19}\right)+1\right]$$

$$N=\left(-\frac{71}{19}\ ;\ 7\ ;\ \frac{34}{19}\right)$$

3) a) L recta ortogonal a las rectas:

$$L_1: \frac{x+6}{2} = y-1 = \frac{z+1}{-1}$$

$$L_2: x-3 = \frac{y}{2} \; ; \; z=2$$

Determine la ecuación del Plano P que pasa por el punto $(0,0,-2)$ y que contiene a la recta L.

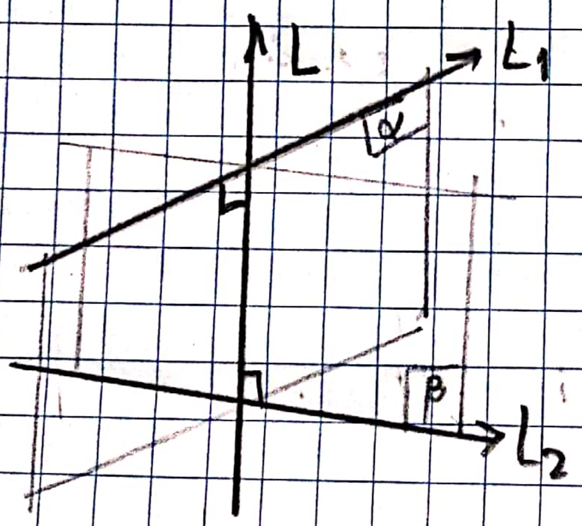
Hallando la ecuación de L:

$$L_1 \begin{cases} P_0 = (-6, 1, -1) \\ V_1 = (2, 1, -1) \end{cases} \qquad L_2 \begin{cases} Q_0 = (3, 0, 2) \\ V_2 = (1, 2, 0) \end{cases}$$

Vector director de L:

$$V = V_1 \times V_2 = (2, 1, -1) \times (1, 2, 0)$$

$$V = (2, -1, 3)$$



$$\pi_\alpha \cap \pi_\beta = L$$

$$\pi_\alpha \begin{cases} P_0 = (-6, 1, -1) \\ V_1 = (2, 1, -1) \\ V = (2, -1, 3) \end{cases}$$

$$\pi_\beta \begin{cases} Q_0 = (3, 0, 2) \\ V_2 = (1, 2, 0) \\ V = (2, -1, 3) \end{cases}$$

$$\Pi_\alpha = \begin{vmatrix} x+6 & y-1 & z+1 \\ 2 & 1 & -1 \\ 2 & -1 & 3 \end{vmatrix} \qquad \Pi_\beta = \begin{vmatrix} x-3 & y & z-2 \\ 1 & 2 & 0 \\ 2 & -1 & 3 \end{vmatrix}$$

$$\Pi_\alpha : 2x+16-8y-4z=0 \qquad \Pi_\beta : 6x-3y-5z-8=0$$

$$\Pi_\alpha \cap \Pi_\beta = \{L\}$$

$$\left\{\begin{array}{l} 2x+16-8y-4z=0 \\ 6x-3y-5z-8=0 \end{array}\right\} \quad \begin{array}{l} x = \frac{2}{3}(z+4) \\ y = \frac{-z+8}{3} \end{array}$$

ECUACIÓN SIMÉTRICA DE L:

$$\frac{3x-8}{2} = \frac{3y-8}{-1} = z$$

$$\frac{x-\frac{8}{3}}{\frac{2}{3}} = \frac{y-\frac{8}{3}}{-\frac{1}{3}} = z$$

$$L = \left\{ \left(\frac{8}{3}, \frac{8}{3}, 0\right) + t\left(\frac{2}{3}, -\frac{1}{3}, 1\right),\ t \in \mathbb{R} \right\}$$

$$Q_0 = (0,0,-2)$$

$$\overrightarrow{P_0Q_0} = Q_0 - P_0 = (0,0,-2) - \left(\frac{8}{3}, \frac{8}{3}, 0\right) = \left(-\frac{8}{3}, -\frac{8}{3}, -2\right)$$

$$V = \left(\frac{2}{3}, -\frac{1}{3}, 1\right)$$

$$N = \overrightarrow{P_0Q_0} \times V = \left(-\frac{8}{3}, -\frac{8}{3}, -2\right) \times \left(\frac{2}{3}, -\frac{1}{3}, 1\right)$$

$$N = \left(\frac{-10}{3}, \frac{4}{3}, \frac{8}{3}\right)$$

Ecuación del plano ($\mathcal{P}$):

$$\mathcal{P}: -\frac{10}{3}(x-0) + \frac{4}{3}(y-0) + \frac{8}{3}(z+2) = 0$$

$$-10x + 4y + 8z + 16 = 0$$

$$\mathcal{P}: \underline{10x - 4y - 8z = 16}$$

b)

i) $(a \times b) \times (b \times c) \cdot (c \times a) = [a\,b\,c]^2$ (V)

$$(a \times b) \times (b \times c) \cdot (c \times a) = (c \times a) \cdot [(a \times b) \times (b \times c)]$$

$$I = (a \times b) \times (b \times c) = ((a \times b) \cdot c)b - (\underbrace{(a \times b) \cdot b}_{0})c$$

$$I = (a \times b) \times (b \times c) = [(a \times b) \cdot c]\,b$$

$$(a \times b) \times (b \times c) \cdot (c \times a) = (c \times a) \cdot [(a \times b) \cdot c]\,b$$

$$= [(a \times b) \cdot c][(c \times a) \cdot b]$$

$$= [a\,b\,c][a\,b\,c]$$

$$(a \times b) \times (b \times c) \cdot (c \times a) = [a\,b\,c]^2$$

$\therefore$ La proposición es verdadera.

ii) Si $\text{proy}_b\, a = (4, 4, -8)$ y $\text{proy}_a\, b = (8, -4, -4)$

entonces $a \cdot b = 64$ (F)

$\text{proy}_b\, a = (4, 4, -8) \longrightarrow b = t(4, 4, -8)$

$\text{proy}_a\, b = (8, -4, -4) \longrightarrow a = m(8, -4, -4)$

$\rightarrow a \cdot b = 32tm - 16tm + 32tm$

$a \cdot b = 48mt$

$\text{proy}_b\, a = \left(\dfrac{a \cdot b}{|b|^2}\right) b = (4, 4, -8)$

PARA LA PRIMERA COMPONENTE

$\rightarrow \left(\dfrac{a \cdot b}{|b|^2}\right) b_1 = 4$

$\dfrac{(48mt)(4t)}{96t^2} = 4$

$\boxed{m = \dfrac{96}{48}}$

$\text{proy}_a\, b = \left(\dfrac{a \cdot b}{|a|^2}\right) a = (8, -4, -4)$

PARA LA PRIMERA COMPONENTE:

$\rightarrow \left(\dfrac{a \cdot b}{|a|^2}\right) a_1 = 8 \rightarrow \dfrac{(48mt)(8m)}{96m^2} = 8 \rightarrow \boxed{t = \dfrac{96}{48}}$

Hallando $a \cdot b$

$a \cdot b = 48mt$

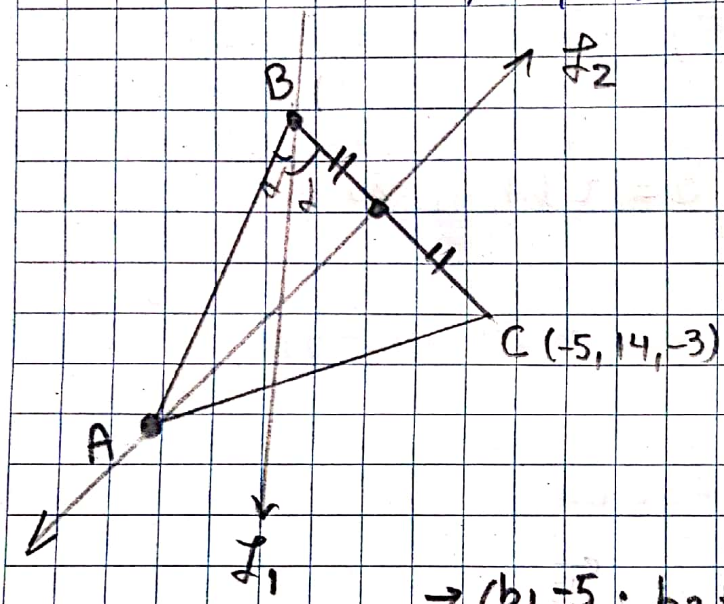
$= 48\left(\dfrac{96}{48}\right)\left(\dfrac{96}{48}\right)$

$= 192$

$\therefore$ La proposición es falsa

4) Dado el ΔABC,

$C = (-5, 14, -3)$



$A = (a_1, a_2, a_3)$
$B = (b_1, b_2, b_3)$

Punto medio de $\overrightarrow{BC} \in L_2$

$$\frac{\overrightarrow{CB}}{2} = \frac{B+C}{2} = \left(\frac{b_1-5}{2}; \frac{b_2+14}{2}; \frac{b_3-3}{2}\right)$$

$$L_2 = \{(-7, 17, -9) + t(-5, 9, -4)\}$$

$$\rightarrow \left(\frac{b_1-5}{2}; \frac{b_2+14}{2}; \frac{b_3-3}{2}\right) = (-7-5t; 9t+17; -4t-9)$$

$$\rightarrow (a_1, a_2, a_3) = (-7-5w; 9w+17; -4w-9)$$

$$L_1 : \{(1, 2, -7) + r(1, -3, -8), r \in \mathbb{R}\}$$

$B \in L_1$:

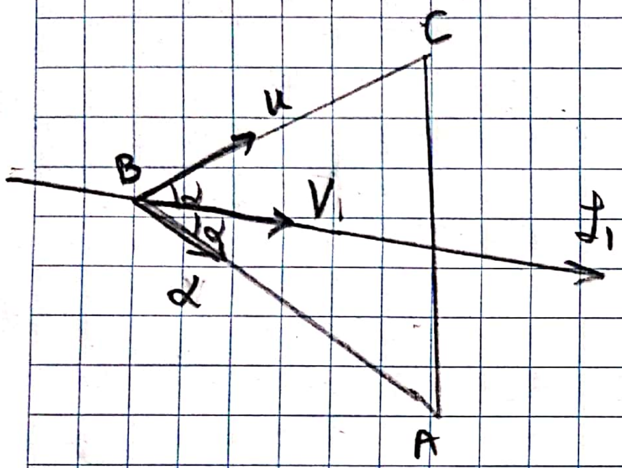
$$(b_1, b_2, b_3) = (1+r, 2-3r, -7-8r)$$

$$\begin{cases} b_1 = -14-10t+5 & b_2 = 18t+34-14 & b_3 = -8t-18+3 \\ b_1 = 1+r & b_2 = 2-3r & b_3 = -7-8r \end{cases}$$

Resolviendo $t = -1$ y $r = 0$

$B = (1, 2, -7)$

Se tiene que:



$\overrightarrow{BC} = \vec{C} - \vec{B} = (-6; 12; 4)$

$\| BC \| = \sqrt{6^2 + 12^2 + 4^2} = 13$

$u_{\overrightarrow{BC}} = \left(\frac{-6}{13}; \frac{12}{13}; \frac{4}{13}\right)$

$\overrightarrow{BA} = \vec{A} - \vec{B} = (a_1 - 1; a_2 - 2; a_3 + 7)$

$\vec{u}_v = \vec{u}_{BC} + \vec{u}_{BA}$; para hallar A.